Introductions

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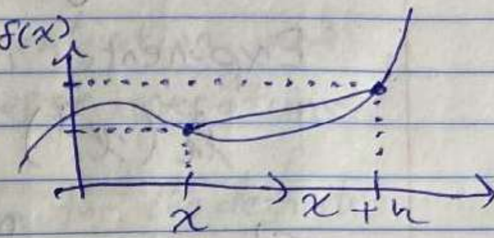
Review of Calculus I

- Derivative

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



- Rules of Differentiation

Polynomials

$$\frac{d}{dx}(x^n) = nx^{n-1} ;$$

Exponentials

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

Sin function

$$\frac{d}{dx}(\sin x) = \cos x$$

Product and Quotient Rules

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

Chain Rule

$$\frac{d}{dx}(f(g(x))) = g'(x) \cdot f'(g(x))$$

- Algebraic Laws of Logarithms

Base

$$\log_b(b) = 1 \quad , \quad \log_b(1) = 0$$

Products and Quotients of logs

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln(x/y) = \ln(x) - \ln(y)$$

Exponents inside logs

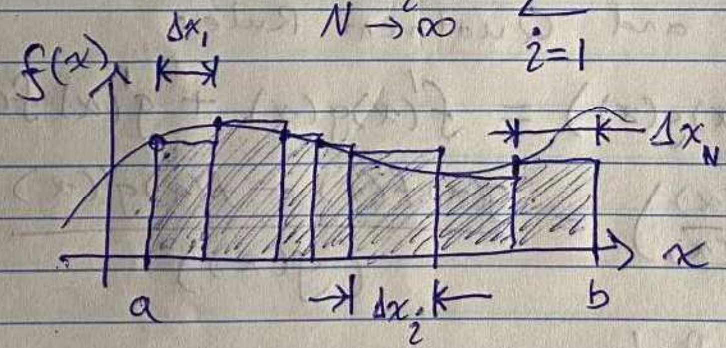
$$\ln(x^y) = y\ln(x)$$

Change of Base Formulae

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

- Integration — as the sum of area under the curve

$$\int_a^b f(x)\,dx = \lim_{\substack{\max \Delta x_i \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \Delta x_i f(x_i^*)$$



- Fundamental Theorem of Calculus

for f, continuous function on an interval $[a,b]$ ~~and differentiable on (a,b)~~ Then

$$F(x) = \int_a^x f(t)\,dt$$

is continuous and differentiable on (a,b)

And furthermore, the derivative of F is given by its integrand

$$F'(x) = f(x)$$

Establishing Differentiation and Integration as Inverse operations!

This yields a formula for the definite integral:
If F is an antiderivative of f
then on an interval [a,b] where f is continuous

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

- Rules for Anti-differentiation

Polynomials

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad \text{unless } n=-1$$

Trigonometric

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

Logarithm as an antiderivative

$$\int x^{-1}\,dx = \ln(x) + C$$

u-Substitution
example

$$I = \int 3x^2\sqrt{x^3+1}\,dx \qquad \text{replace } u = x^3+1$$

then $du = 3x^2\,dx$

$$= \int \sqrt{u}\,du$$

expression has been simplified

example

$$I = \int_{\pi/4}^{\pi/2} \cot\theta \csc^2\theta \, d\theta$$

let $u = \cot\theta$, then $du = -\csc^2\theta \, d\theta$

Proceed by transforming the limits of integration

θ	$u = \cot\theta$	
$\pi/4$	$\cot(\pi/4) =$	1
$\pi/2$	$\cot(\pi/2) =$	0

The integral becomes

$$I = \int_1^0 u(-du) = \int_0^1 u\,du$$

$$= \left(\tfrac{1}{2}u^2\right)\Big|_{u=0}^{1}$$

$$= \tfrac{1}{2} \checkmark$$

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